Machine Learning I

MICRO-455

Classification with SVM

The Teaching Team

Fall 2024

Q1

- Computational Aspects of SVM.
  - Number of datapoints: M.
  - Data dimension: N.
  - Number of support vectors: S.

Q1 > A

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- It is possible to store one less parameter; but it is not practical.

$$\sum_i^M \alpha_i y_i = \sum_i^S \alpha_i y_i = 0$$

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$$\boxed{\frac{1,010,000 \times 8\text{B}}{1024 \times 1024} \approx 7.71\text{MB}}$$

Q1 > C

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- What is training time for M=1,000,000 and N=100?
- The training time complexity is assumed to be: $\mathcal{O}(MN^2)$.

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$$\boxed{100,000 \times 0.1\text{s} = 10,000\text{s} = 166\frac{2}{3}\text{min} \approx 2.78\text{h} \approx 2\text{h}47\text{min}}$$

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$$\boxed{2.78\mathrm{h} \times 50\mathrm{W} = 139\mathrm{Wh}}$$

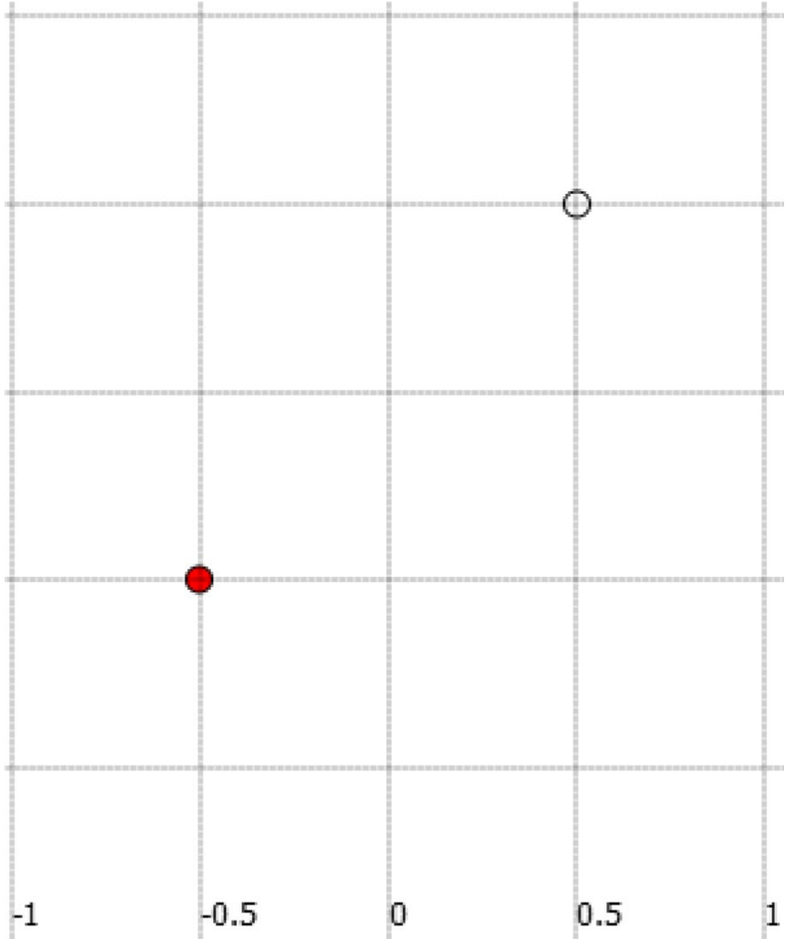
$$\boxed{\frac{5}{60}\mathrm{h} \times 1500\mathrm{W} = 125\mathrm{Wh}}$$

Q2 > A

- Compute the coefficients and the bias term of the SVM classifier for two datapoints with RBF Kernel.
- 1: X=[0.5, 0.5]; y=1
- 2: X=[-0.5, -0.5]; y=-1

$$\boxed{k(\boldsymbol{x}, \boldsymbol{x}_i) = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{x}_i\|^2}{2\sigma^2}\right)}$$

- k(X1, X2) = 0.5 = k(X2, X1)
- k(X1, X1) = 1 = k(X2, X2)



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$$\boxed{\sum_i \alpha_i y_i = 0 \longrightarrow \alpha_1 = \alpha_2}$$

$$\boxed{\alpha_1 = \alpha_2 = 2; \quad b = 0}$$

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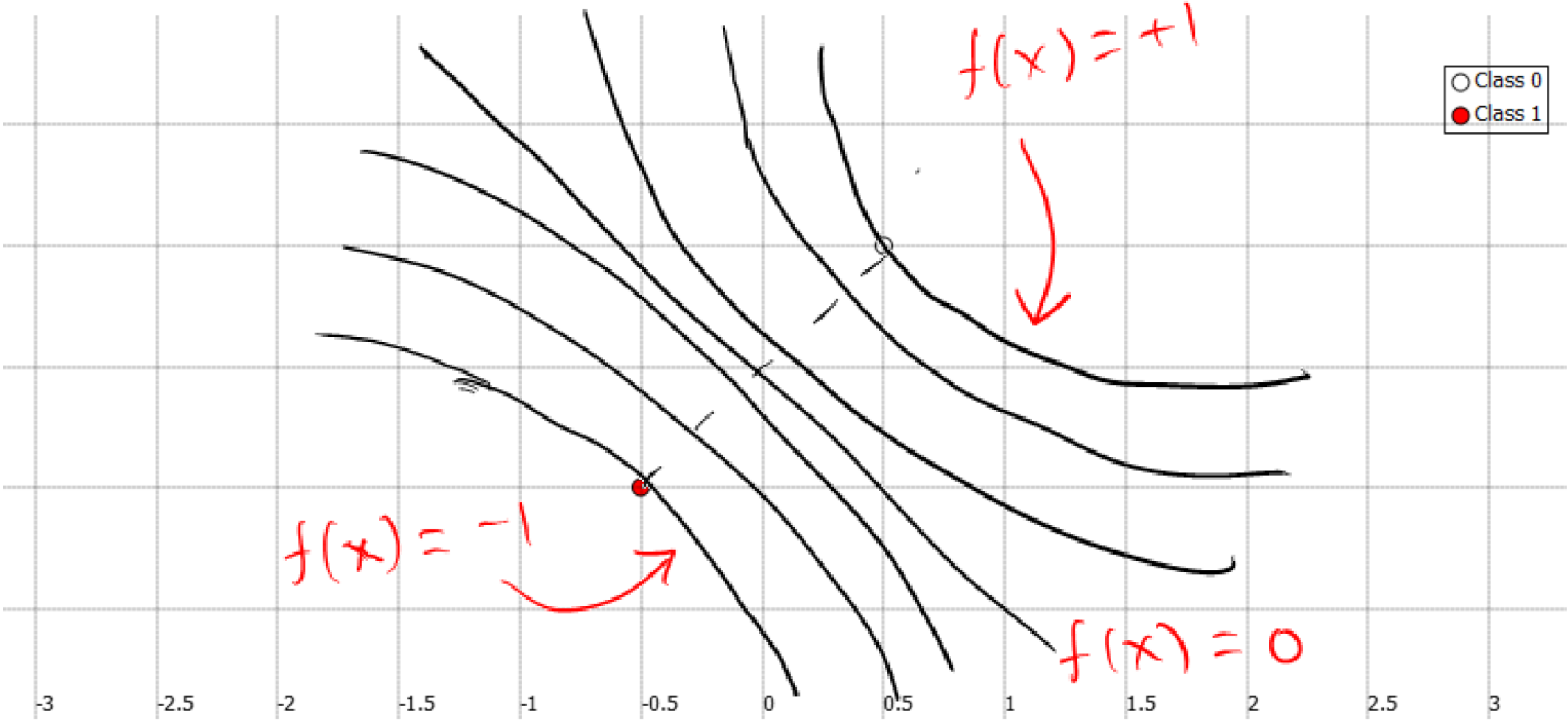
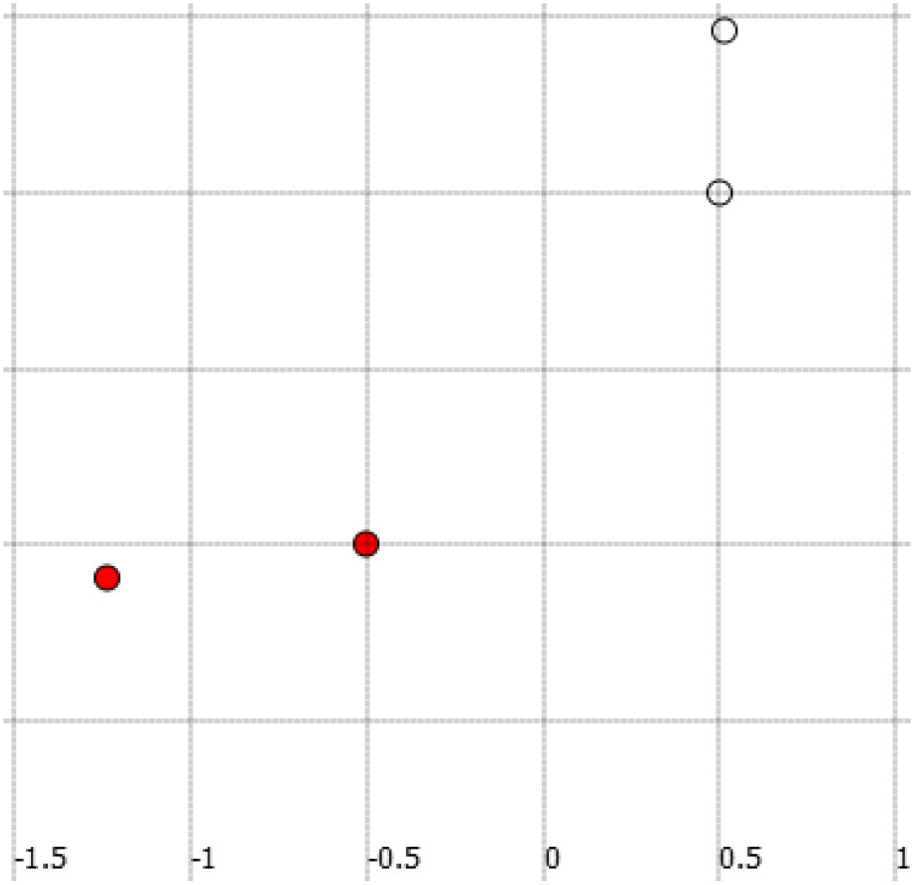


Q2 > B – Case 1

- Draw the new separating hyperplane.



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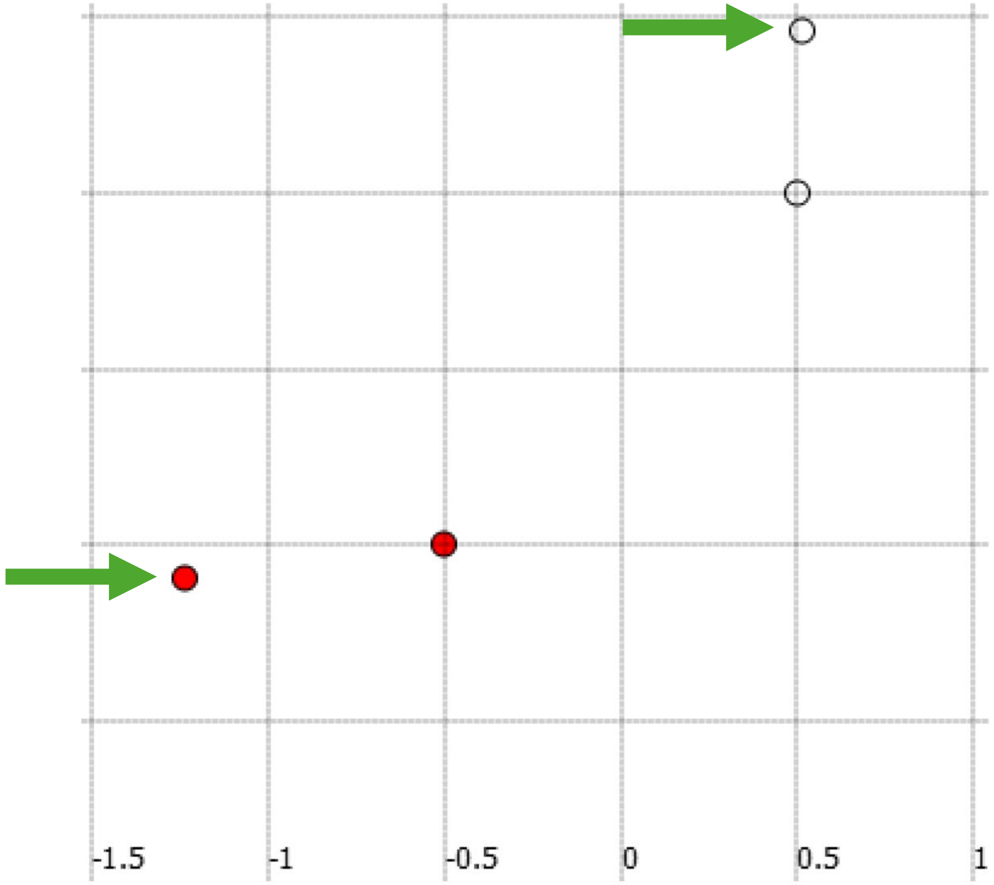

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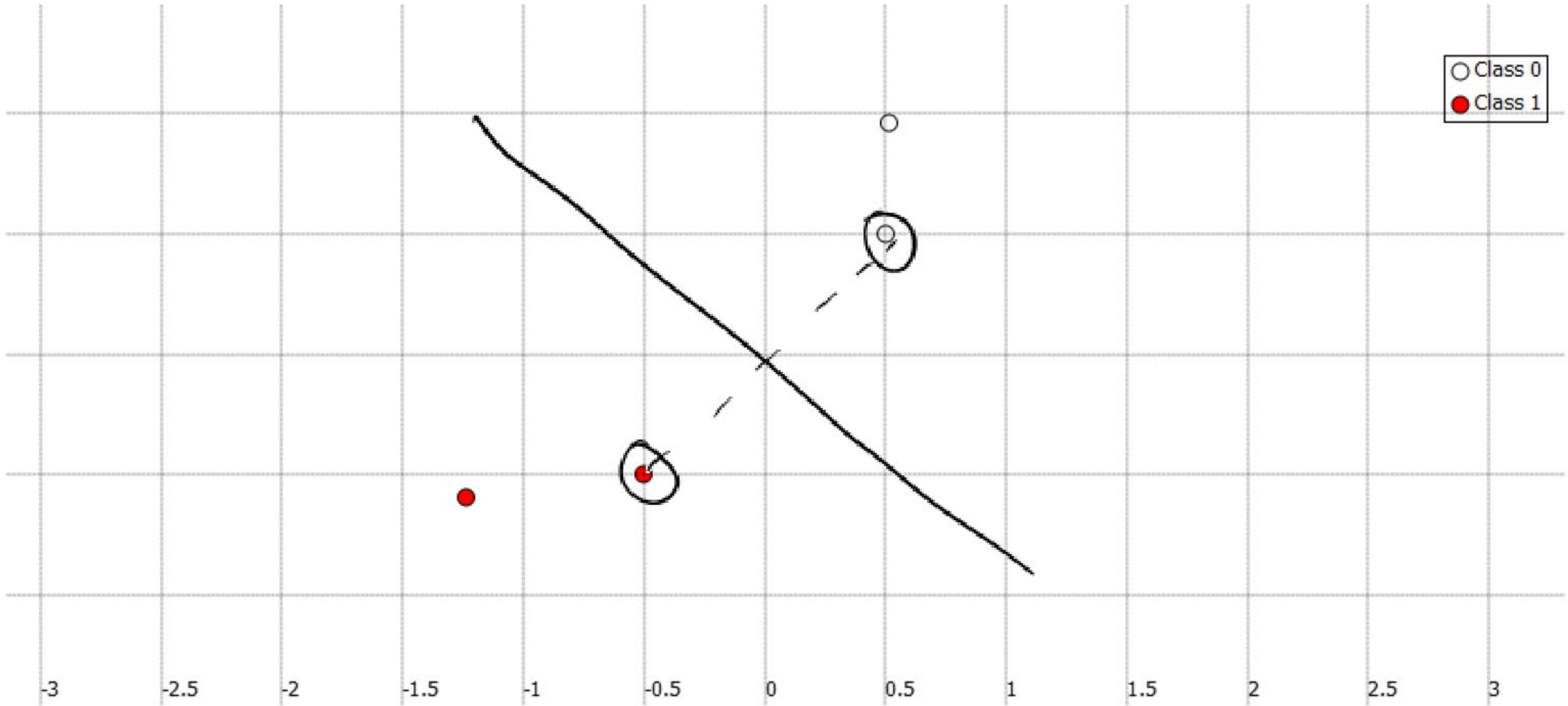
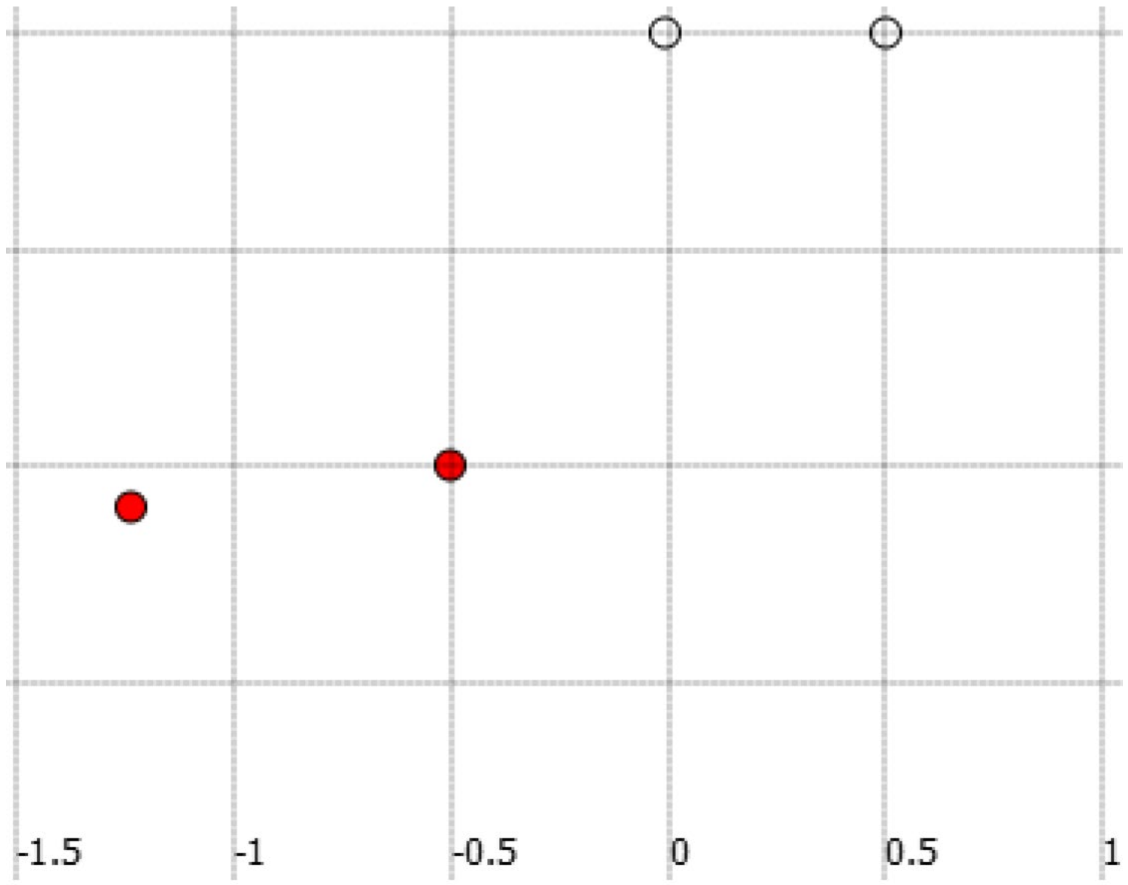


Q2 > B – Case 2

- Draw the new separating hyperplane.



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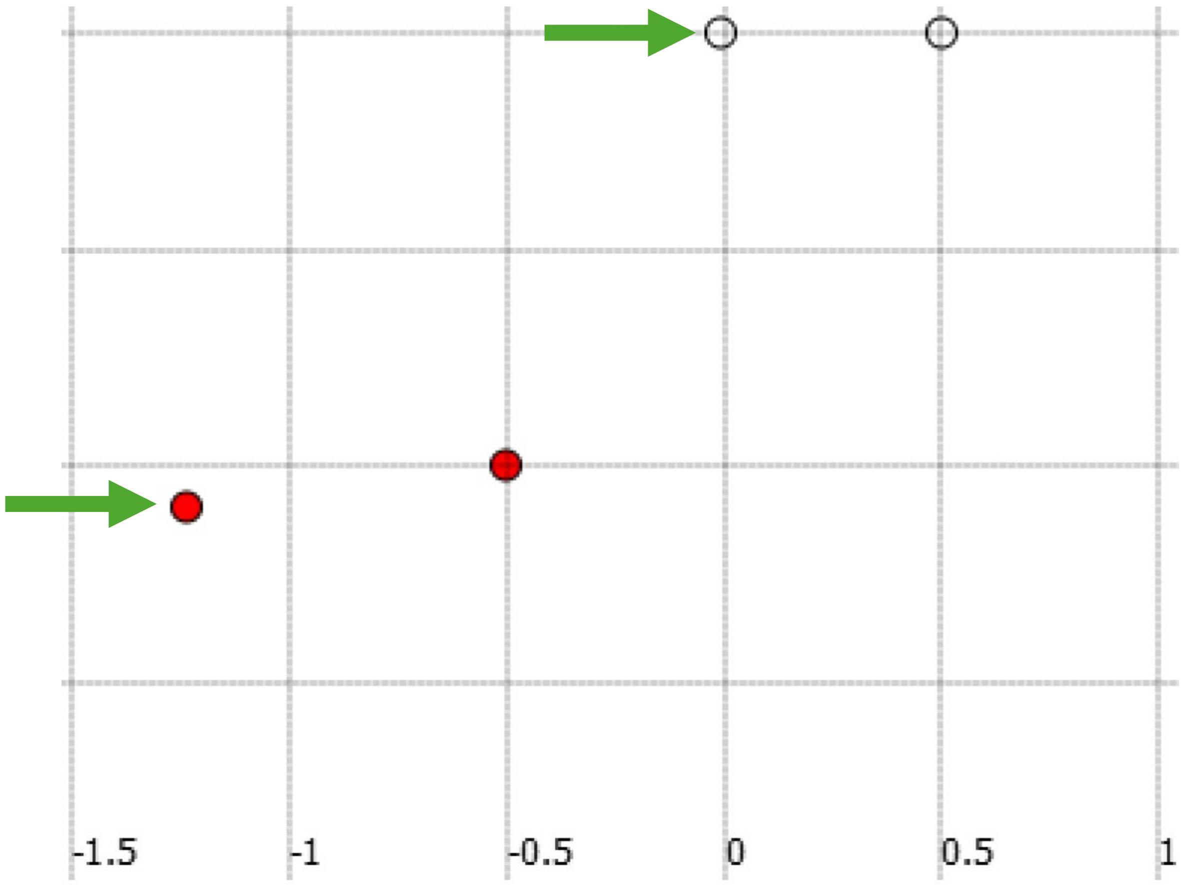

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- Since the point added to the white class is inside the original margin, it now becomes a support vector instead the original point in the white class.

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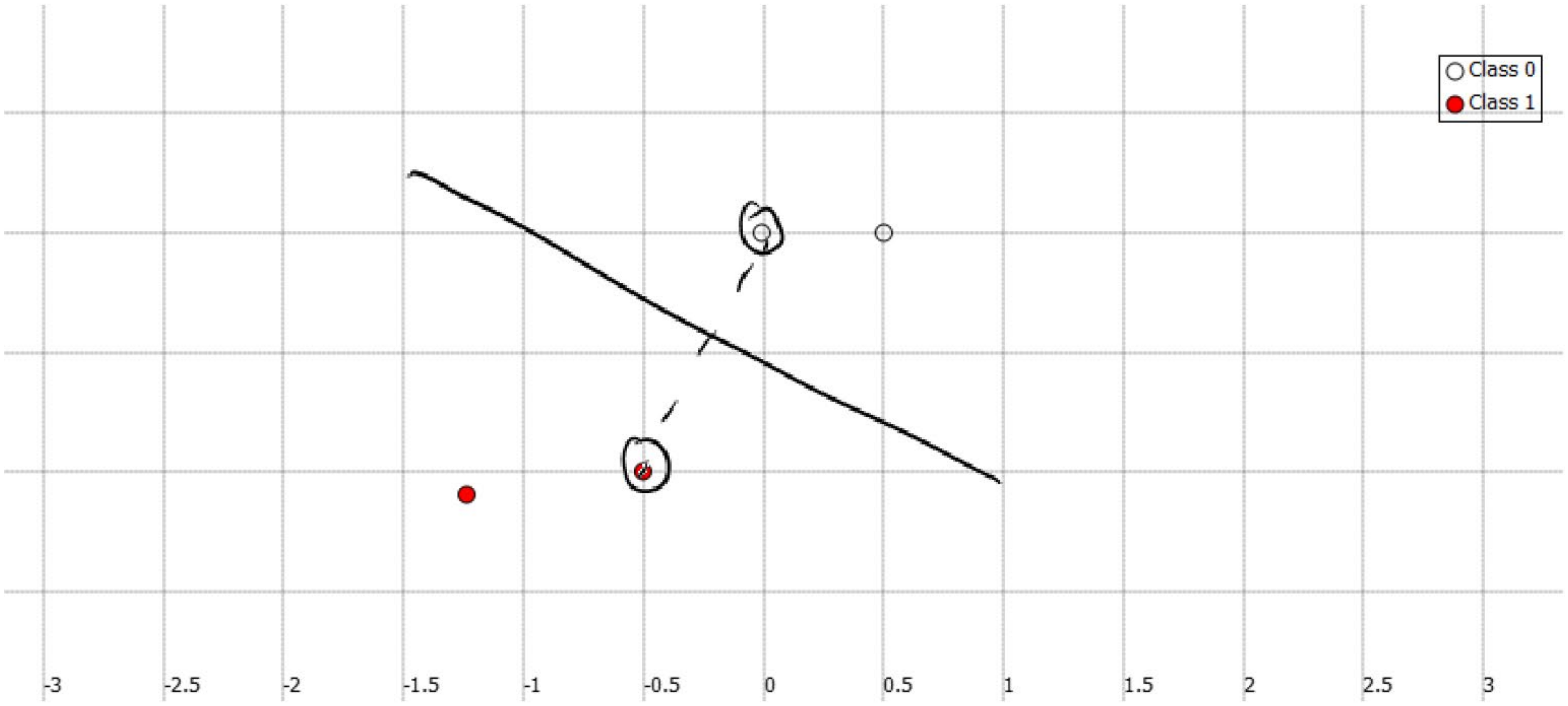
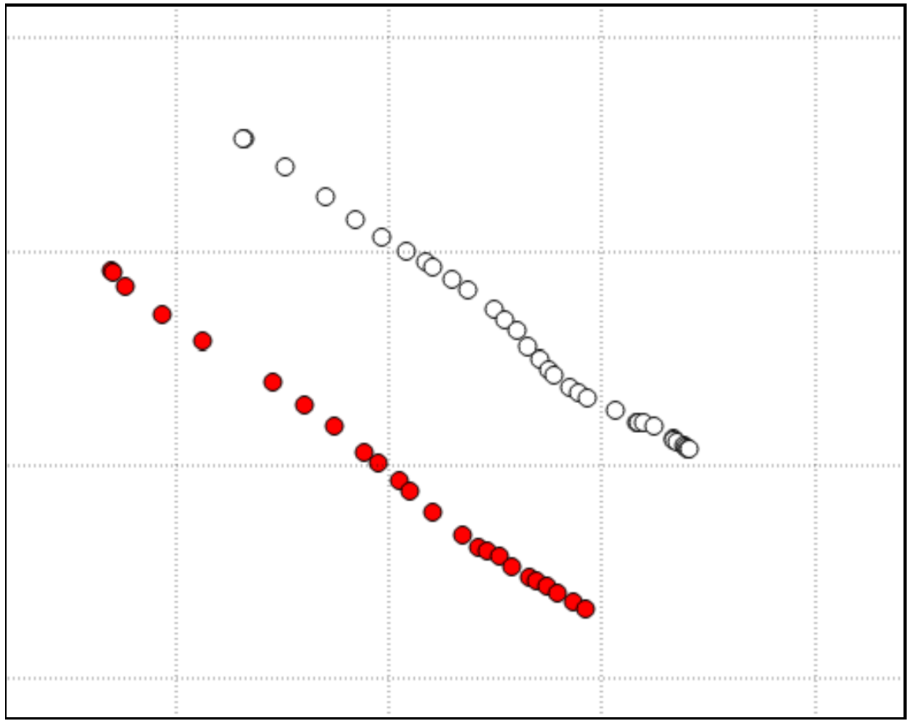
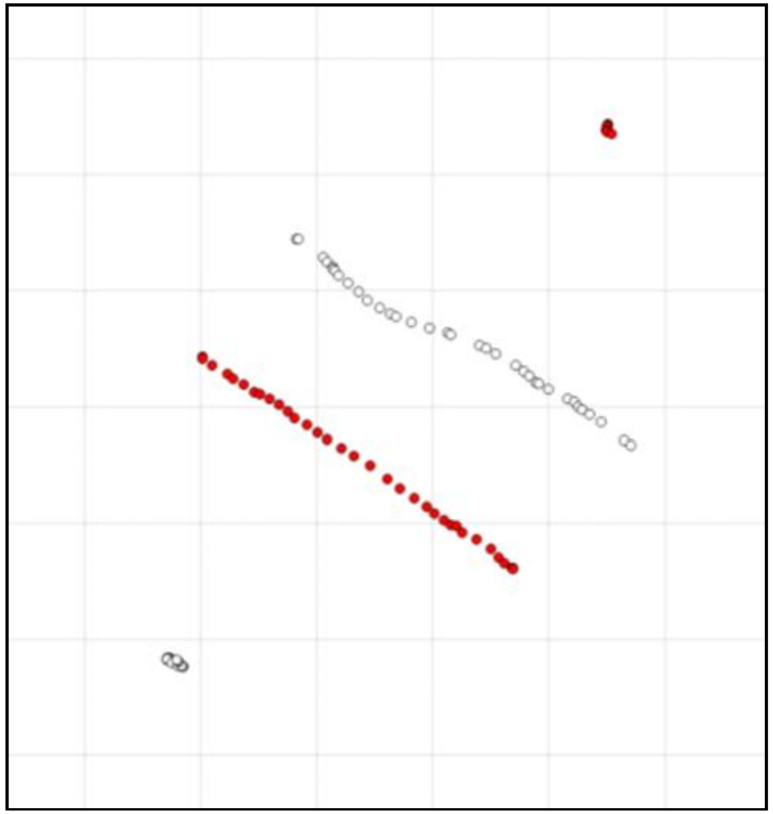


Q2 > C

- Draw the separating hyperplane and discuss the associated effects of the values of the hyperparameters (penalty factor and kernel width).



(i)



(ii)

Q2 > C – Case i

- The separating line is unaffected by the value of the penalty C.
- The kernel width affects the number of support vectors.

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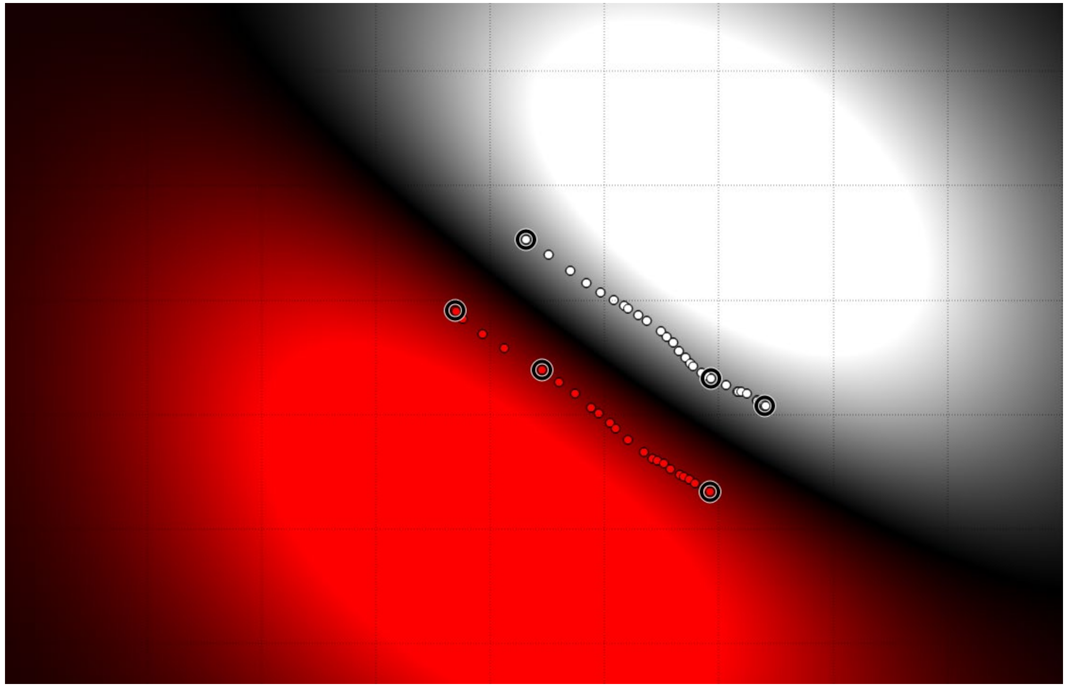
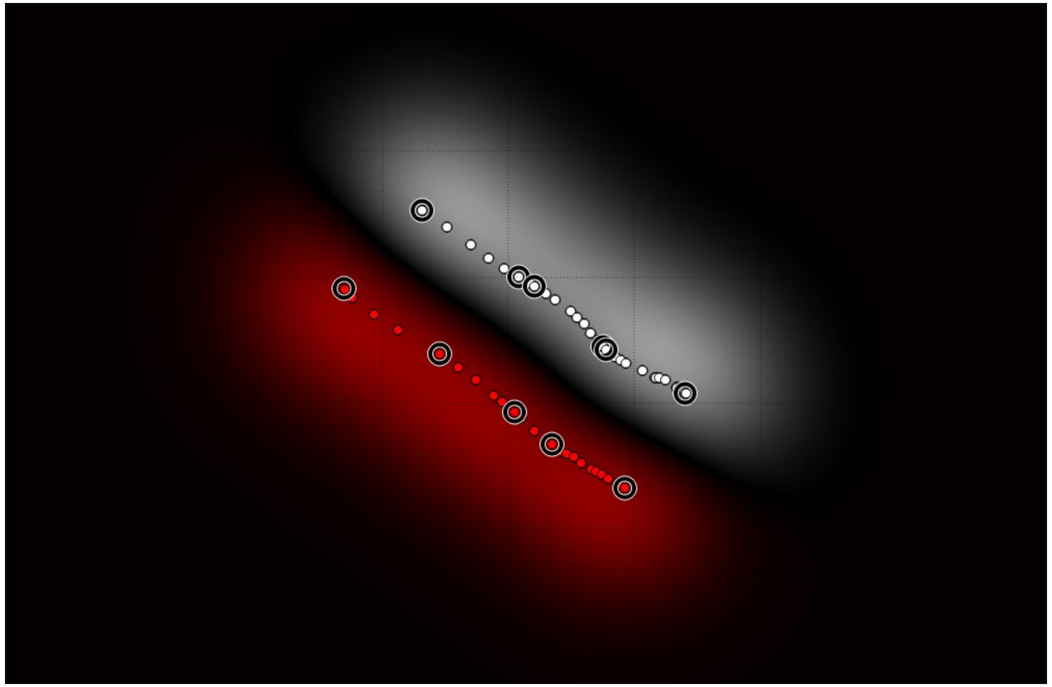
$\sigma = 0.1$



$\sigma = 0.01$



Q2 > C – Case ii

- The separating line will be influenced by both penalty C and the kernel width.

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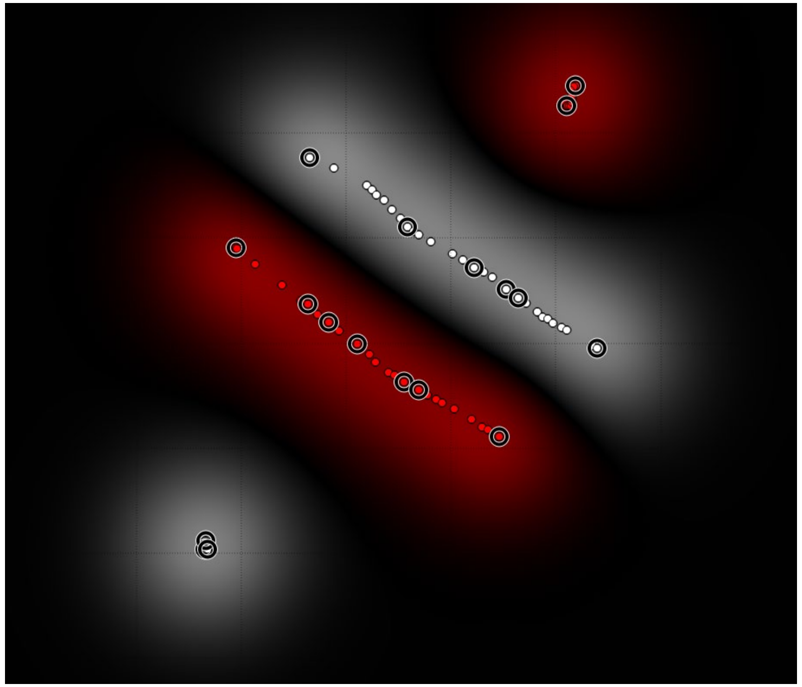
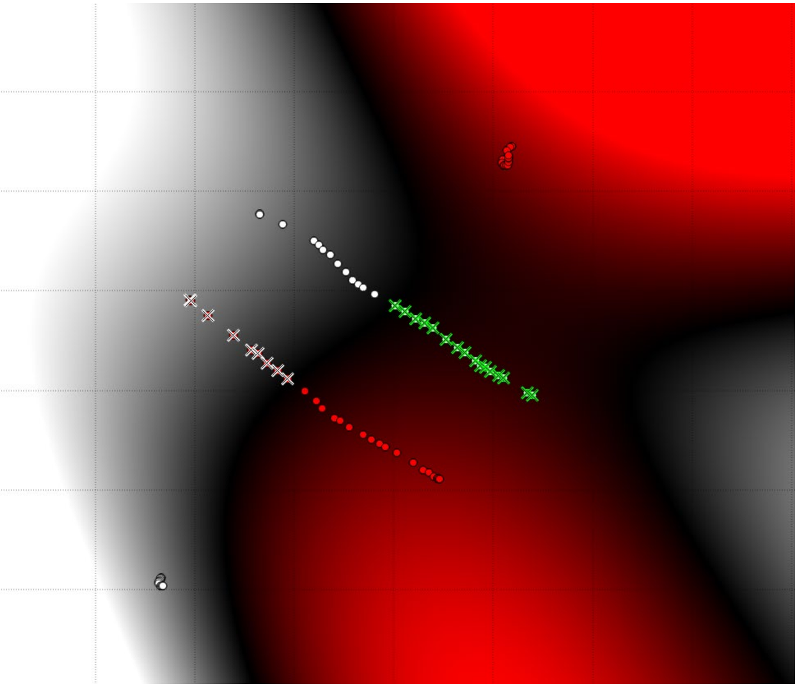
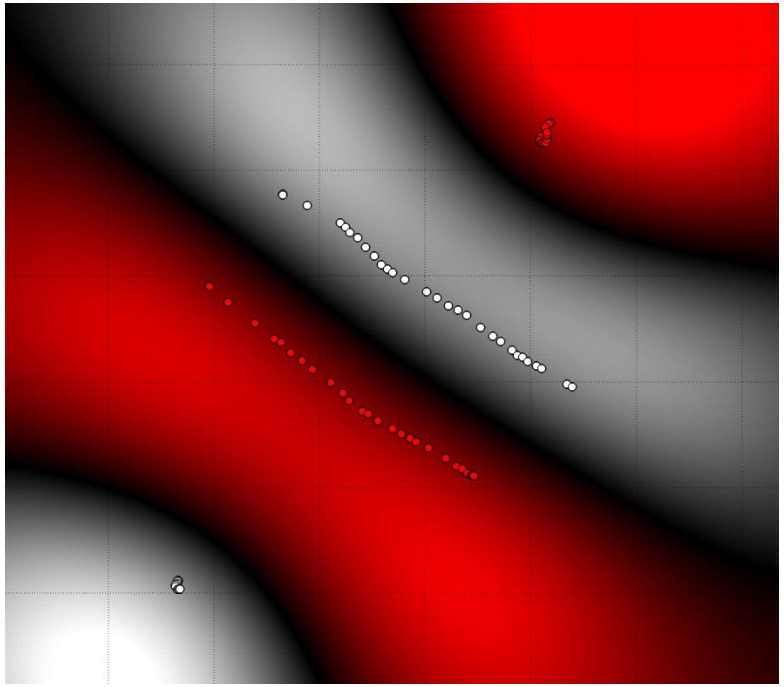
$\sigma = 0.01; C = 5000$



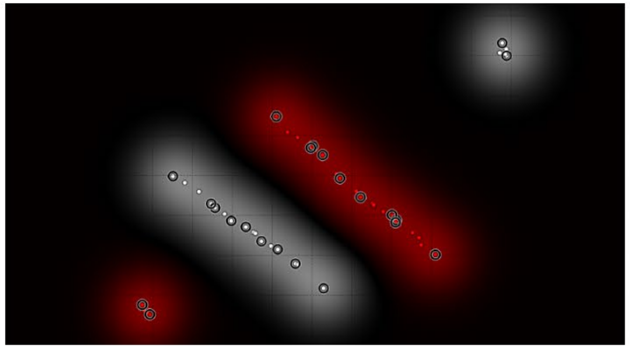
$\sigma = 0.5; C = 10$



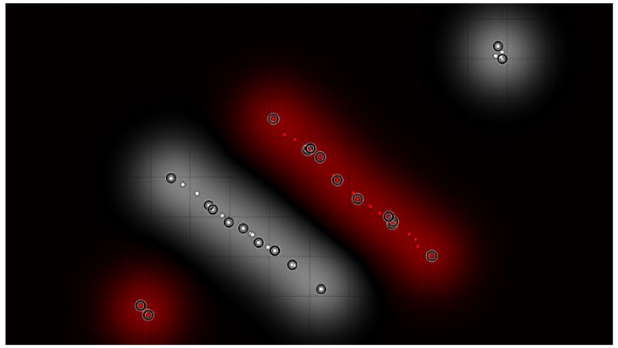
$\sigma = 0.1; C = 1000$



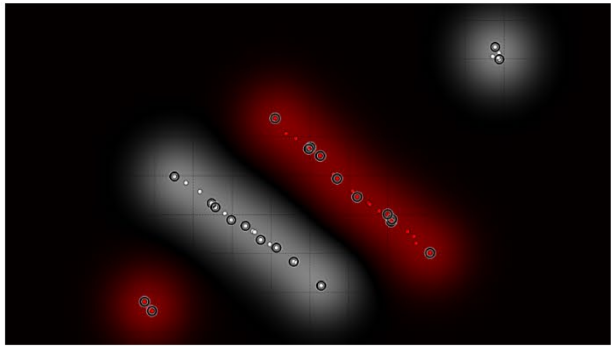
Q2 > C – Case ii



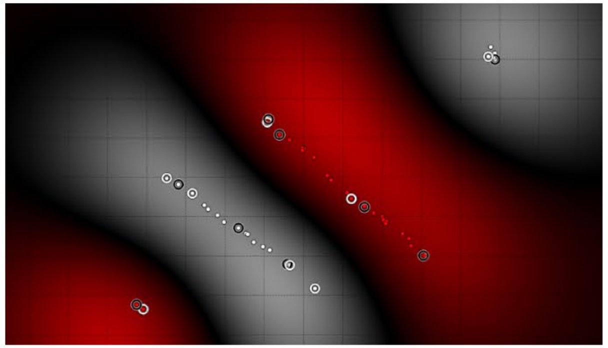
(a) $\sigma = 0.01; C = 1$



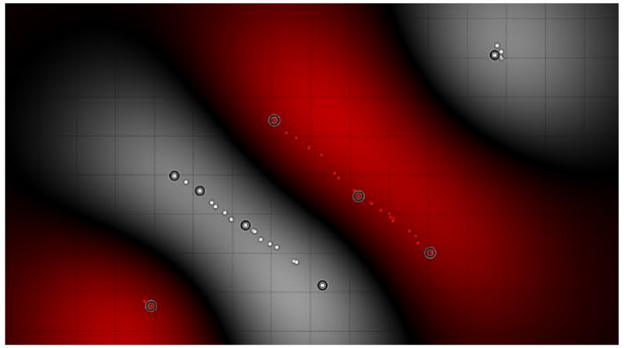
(b) $\sigma = 0.01; C = 10$



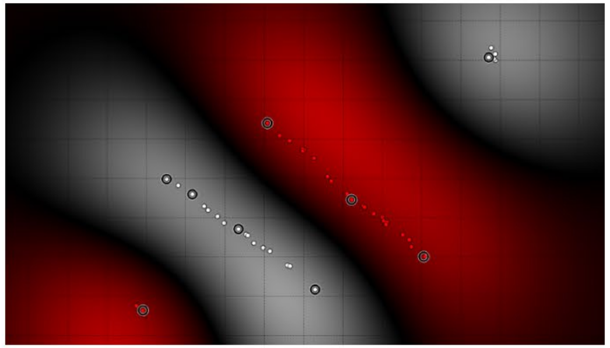
(c) $\sigma = 0.01; C = 5000$



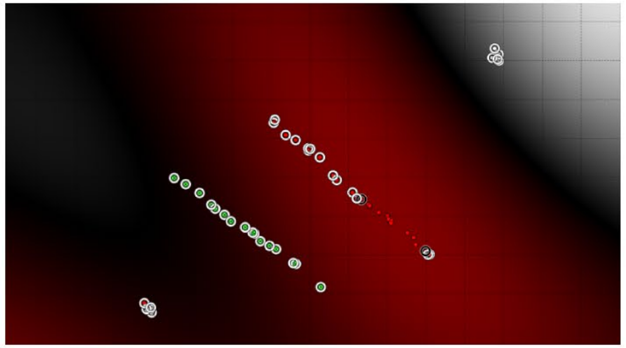
(d) $\sigma = 0.1; C = 1$



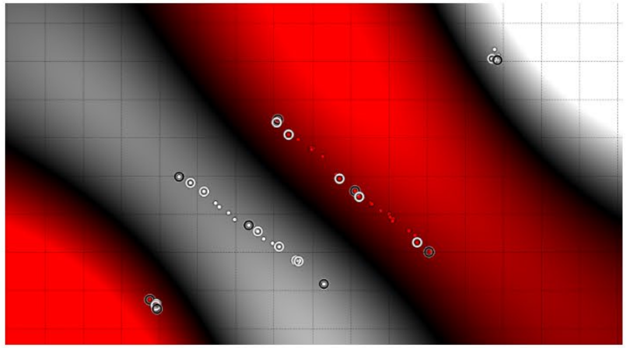
(e) $\sigma = 0.1; C = 10$



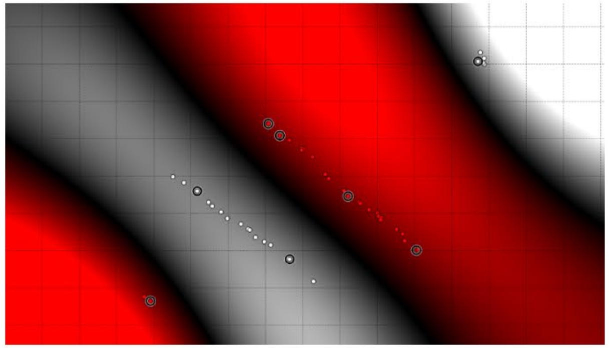
(f) $\sigma = 0.1; C = 5000$



(g) $\sigma = 0.5; C = 1$



(h) $\sigma = 0.5; C = 10$



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